

Nonclassical correlations of photon number and field components in the vacuum state

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It is shown that the quantum jumps in the photon number \hat{n} from zero to one or more photons induced by backaction evasion quantum nondemolition measurements of a quadrature component \hat{x} of the vacuum light field state are strongly correlated with the quadrature component measurement results. This correlation corresponds to the operator expectation value $\langle \hat{x}\hat{n}\hat{x} \rangle$ which is equal to one-fourth for the vacuum even though the photon number eigenvalue is zero. Quantum nondemolition measurements of a quadrature component can thus provide experimental evidence of the nonclassical operator ordering dependence of the correlations between photon number and field components in the vacuum state.

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I. INTRODUCTION

One of the main differences between quantum mechanics and classical physics is the impossibility of assigning well-defined values to all physical variables describing a system. As a consequence, all quantum measurements necessarily introduce noise into the system. A measurement which only introduces noise in those variables that do not commute with the measured variable is referred to as a quantum nondemolition (QND) measurement [1]. In most of the theoretical and experimental investigations [2–8], the focus has been on the overall measurement resolution and on the reduction of fluctuations in the QND variable as observed in the correlation between the QND measurement results and a subsequent destructive measurement of the QND variable. However, at finite resolution, quantum nondemolition measurements do not completely destroy the original coherence between eigenstates of the QND variable [9,10]. By correlating the QND measurement result with subsequent destructive measurements of a noncommuting variable, it is therefore possible to determine details of the measurement induced decoherence [11].

In particular, QND measurements of a quadrature component of the light field introduce not only noise in the conjugated quadrature component, but also in the photon number of a state. By measuring a quadrature component of the vacuum field, “quantum jumps” from zero photons to one or more photons are induced in the observed field. It is shown in the following that, even at low measurement resolutions, the “quantum jump” events are strongly correlated with extremely high measurement results for the quadrature component. This correlation corresponds to a nonclassical relationship between the continuous field components and the discrete photon number, which is directly related to fundamental properties of the operator formalism. Thus, this experimentally observable correlation of photon number and fields reveals important details of the physical meaning of quantization.

In Sec. II QND measurements of a quadrature component

\hat{x} of the light field are discussed and a general measurement operator $\hat{P}_{\delta x}(x_m)$ describing a minimum noise measurement at a resolution of δx is derived. In Sec. III the measurement operator is applied to the vacuum field and the measurement statistics are determined. In Sec. IV the results are compared with fundamental properties of the operator formalism. In Sec. V an experimental realization of photon-field coincidence measurements is proposed and possible difficulties are discussed. In Sec. VI the results are interpreted in the context of quantum state tomography and implications for the interpretation of entanglement are pointed out. In Sec. VII the results are summarized and conclusions are presented.

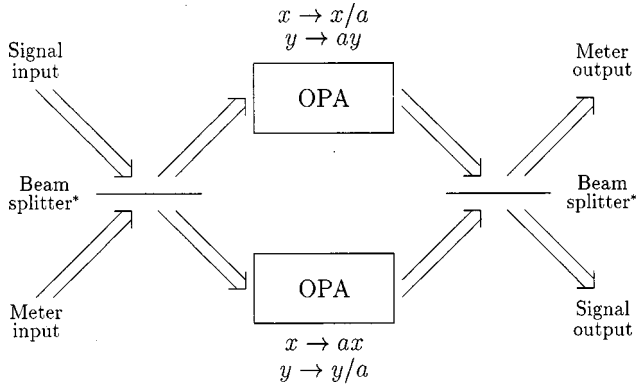
II. QND MEASUREMENT OF A QUADRATURE COMPONENT

Optical QND measurements of the quadrature component \hat{x}_S of a signal mode $\hat{a}_S = \hat{x}_S + i\hat{y}_S$ are realized by coupling the signal to a meter mode $\hat{a}_M = \hat{x}_M + i\hat{y}_M$ in such a way that the quadrature component \hat{x}_M of the meter mode is shifted by an amount proportional to the measured signal variable \hat{x}_S . This measurement interaction can be described by a unitary transformation operator,

$$\hat{U}_{SM} = \exp(-i2f\hat{x}_S\hat{y}_M), \quad (1)$$

which transforms the quadrature components of meter and signal to

$$\begin{aligned} \hat{U}_{SM}^{-1}\hat{x}_S\hat{U}_{SM} &= \hat{x}_S, \\ \hat{U}_{SM}^{-1}\hat{y}_S\hat{U}_{SM} &= \hat{y}_S - f\hat{y}_M, \\ \hat{U}_{SM}^{-1}\hat{x}_M\hat{U}_{SM} &= \hat{x}_M + f\hat{x}_S, \\ \hat{U}_{SM}^{-1}\hat{y}_M\hat{U}_{SM} &= \hat{y}_M. \end{aligned} \quad (2)$$



$$* \text{ beam splitter reflectivity } R = \frac{a^2}{a^2+1}$$

FIG. 1. Schematic illustration of the measurement setup for a back action evasion quantum nondemolition measurement of a quadrature component using optical parametric amplifiers (OPAs). Note that the reflectivity of the beam splitters depends on the amplification achieved in the parametric down-conversion process. The coupling factor for the measurement is given by $f=(a^2-1)/a$.

In general, the unitary measurement interaction operator \hat{U}_{SM} creates entanglement between the signal and the meter by correlating the values of the quadrature components. Such an entanglement can be realized experimentally by squeezing the two mode light field of the signal and meter using optical parametric amplifiers (OPAs) [6–8]. The measurement setup is shown schematically in Fig. 1. Note that the backaction changing \hat{x}_S is avoided by adjusting the interference between the two amplified beams. Therefore, the reflectivity of the beam splitters depends on the amplification. A continuous adjustment of the coupling factor f would require adjustments of both the pump beam intensities of the OPAs and the reflectivities of the beam splitter as given in Fig. 1.

If the input state of the meter is the vacuum field state, $|\text{vac}\rangle$, and the signal field state is given by $|\Phi_S\rangle$, then the entangled state created by the measurement interaction is given by

$$\begin{aligned} \hat{U}_{SM}|\Phi_S; \text{vac}\rangle &= \int dx_S dx_M \langle x_S | \Phi_S \rangle \langle x_M - f x_S | \text{vac} \rangle |x_S; x_M\rangle \\ &= \int dx_S dx_M \left(\frac{2}{\pi}\right)^{1/4} \exp[-(x_M - f x_S)^2] \\ &\quad \times \langle x_S | \Phi_S \rangle |x_S; x_M\rangle. \end{aligned} \quad (3)$$

Reading out the meter variable x_M removes the entanglement by destroying the coherence between states with different x_M . It is then possible to define a measurement operator $\hat{P}_f(x_M)$ associated with a readout of x_M , which acts only on the initial signal state $|\Phi_S\rangle$. This operator is given by

$$\begin{aligned} \langle x_S | \hat{P}_f(x_M) | \Phi_S \rangle &= \langle x_S; x_M | \hat{U}_{SM} | \Phi_S; \text{vac} \rangle \\ &= \left(\frac{2}{\pi}\right)^{1/4} \exp[-(x_M - f x_S)^2] \langle x_S | \Phi_S \rangle. \end{aligned} \quad (4)$$

The measurement operator $\hat{P}_f(x_M)$ multiplies the probability amplitudes of the \hat{x}_S eigenstates with a Gaussian statistical weight factor given by the difference between the eigenvalue x_S and the measurement result x_M/f . By defining

$$\begin{aligned} x_m &= \frac{1}{f} x_M, \\ \delta x &= \frac{1}{2f}, \end{aligned} \quad (5)$$

the measurement readout can be scaled, so that the average results correspond to the expectation value of \hat{x}_S . The normalized measurement operator then reads

$$\hat{P}_{\delta x}(x_m) = (2\pi\delta x^2) \exp\left(-\frac{(x_m - \hat{x}_S)^2}{4\delta x^2}\right). \quad (6)$$

This operator describes an ideal quantum nondemolition measurement of finite resolution δx . The probability distribution of the measurement results x_m is given by

$$\begin{aligned} P(x_m) &= \langle \Phi_S | \hat{P}_{\delta x}^2(x_m) | \Phi_S \rangle \\ &= \frac{1}{\sqrt{2\pi\delta x^2}} \int dx_S \exp\left(-\frac{(x_S - x_m)^2}{2\delta x^2}\right) |\langle x_S | \Phi_S \rangle|^2. \end{aligned} \quad (7)$$

Thus the probability distribution of measurement results is equal to the convolution of $|\langle x_S | \Phi_S \rangle|^2$ with a Gaussian of variance δx . The corresponding averages of x_m and x_m^2 are given by

$$\int dx_S x_m P(x_m) = \langle \Phi_S | \hat{x}_S | \Phi_S \rangle, \quad (8)$$

$$\int dx_S x_m^2 P(x_m) = \langle \Phi_S | \hat{x}_S^2 | \Phi_S \rangle + \delta x^2.$$

The measurement readout x_m therefore represents the actual value of \hat{x}_S within an error margin of $\pm \delta x$. The signal state after the measurement is given by

$$|\phi_S(x_m)\rangle = \frac{1}{\sqrt{P(x_m)}} \hat{P}_{\delta x}(x_m) |\Phi_S\rangle. \quad (9)$$

Since the quantum coherence between the eigenstates of \hat{x}_S is preserved, the system state is still a pure state after the measurement. The system properties which do not commute with \hat{x}_S are changed by the modified statistical weight of each eigenstate component. Thus the physical effect of noise in the measurement interaction is correlated with the measurement information obtained.

III. MEASUREMENT OF THE VACUUM FIELD

If the signal is in the vacuum state $|\text{vac}\rangle$, then the measurement probability is a Gaussian centered around $x_m=0$ with a variance of $\delta x^2+1/4$,

$$P(x_m) = \frac{1}{\sqrt{2\pi(\delta x^2+1/4)}} \exp\left(-\frac{x_m^2}{2(\delta x^2+1/4)}\right). \quad (10)$$

The quantum state after the measurement is a squeezed state given by

$$\begin{aligned} |\phi_S(x_m)\rangle &= \int dx_S \left(\pi \frac{4\delta x^2}{1+4\delta x^2} \right)^{-1/4} \\ &\times \exp\left[-\frac{1+4\delta x^2}{4\delta x^2} \left(x_S - \frac{x_m}{1+4\delta x^2}\right)^2\right] |x_S\rangle. \end{aligned} \quad (11)$$

The quadrature component averages and variances of this state are

$$\begin{aligned} \langle \hat{x}_S \rangle_{x_m} &= \frac{x_m}{1+4\delta x^2}, \\ \langle \hat{y}_S \rangle_{x_m} &= 0, \end{aligned} \quad (12)$$

$$\begin{aligned} \langle \hat{x}_S^2 \rangle_{x_m} - \langle \hat{x}_S \rangle_{x_m}^2 &= \frac{\delta x^2}{1+4\delta x^2}, \\ \langle \hat{y}_S^2 \rangle_{x_m} - \langle \hat{y}_S \rangle_{x_m}^2 &= \frac{1+4\delta x^2}{16\delta x^2}. \end{aligned}$$

Examples of the phase-space contours before and after the measurement are shown in Fig. 2 for a measurement resolution of $\delta x=0.5$ and a measurement result of $x_m=-0.5$. Note that the final state is shifted by only half the measurement result.

The photon number expectation value after the measurement is given by the expectation values of \hat{x}_S^2 and \hat{y}_S^2 . It reads

$$\begin{aligned} \langle \hat{n}_S \rangle_{x_m} &= \langle \hat{x}_S^2 \rangle_{x_m} + \langle \hat{y}_S^2 \rangle_{x_m} - \frac{1}{2} \\ &= \frac{1}{16\delta x^2(1+4\delta x^2)} + \frac{x_m^2}{(1+4\delta x^2)^2}. \end{aligned} \quad (13)$$

The dependence of the photon number expectation value $\langle \hat{n}_S \rangle_{x_m}$ after the measurement on the squared measurement result x_m^2 describes a correlation between field component and photon number defined by

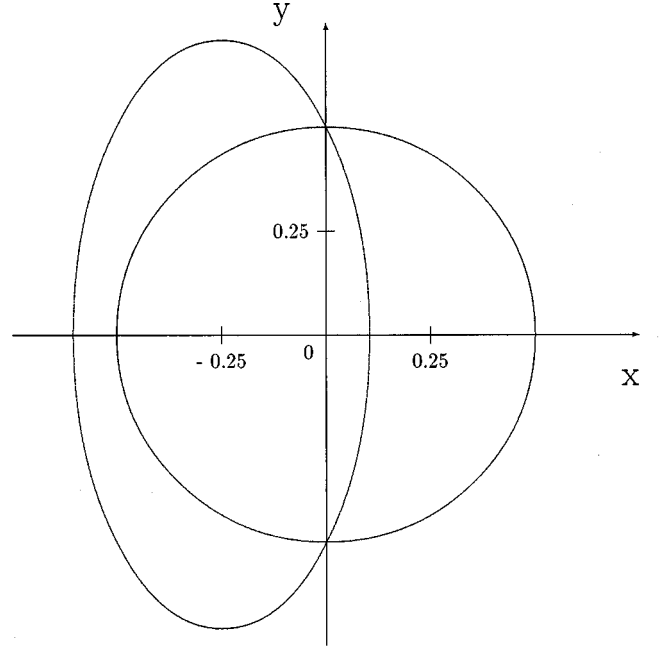


FIG. 2. Visualization of the field fluctuations before and after the measurement for a measurement resolution of $\delta x=0.5$ and a measurement result of $x_m=-0.5$. The contours shown mark the standard deviation of the Gaussian noise distributions. The circle represents the vacuum fluctuations. After the measurement, the x component is shifted by $x_m/2=-0.25$ and the fluctuations in x are squeezed by a factor of $1/\sqrt{2}$. The fluctuations in y are increased by a factor of $\sqrt{2}$ by the noise introduced in the measurement.

$$\begin{aligned} C(x_m^2; \langle \hat{n}_S \rangle_{x_m}) &= \int \left(\int dx_m x_m^2 \langle \hat{n}_S \rangle_{x_m} P(x_m) \right) \\ &- \left(\int dx_m x_m^2 P(x_m) \right) \\ &\times \left(\int dx_m \langle \hat{n}_S \rangle_{x_m} P(x_m) \right). \end{aligned} \quad (14)$$

According to Eqs. (10) and (13), this correlation is equal to

$$C(x_m^2; \langle \hat{n}_S \rangle_{x_m}) = \frac{1}{8} \quad (15)$$

for measurements of the vacuum state. This result is independent of the measurement resolution. In particular, it even applies to the low resolution limit of $\delta x \rightarrow \infty$, which should leave the original vacuum state nearly unchanged. It is therefore reasonable to conclude that this correlation is a fundamental property of the vacuum state, even though it involves nonzero photon numbers.

IV. CORRELATIONS OF PHOTON NUMBER AND FIELDS IN THE OPERATOR FORMALISM

Since the measurement readout x_m represents information about operator variable \hat{x}_S of the system, it is possible to express the correlation $C(x_m^2; \langle \hat{n}_S \rangle_{x_m})$ in terms of operator

expectation values of \hat{x}_S and \hat{n}_S . Equation (8) shows how the average over x_m^2 can be replaced by the operator expectation value $\langle \hat{x}_S^2 \rangle$. Likewise, the average over the product of x_m^2 and $\langle \hat{n}_S \rangle_{x_m}$ can be transformed into an operator expression. The transformation reads

$$\begin{aligned} & \int dx_m x_m^2 \langle \hat{n}_S \rangle_{x_m} P(x_m) \\ &= \int dx_S dx'_S \left(\frac{(x_S + x'_S)^2}{4} + \delta x^2 \right) \langle \text{vac} | x_S \rangle \langle x_S | \hat{n}_S | x'_S \rangle \\ & \quad \times \langle x'_S | \text{vac} \rangle \exp \left(-\frac{(x_S - x'_S)^2}{8 \delta x^2} \right) \\ &= \int dx_m \left(\frac{1}{4} \langle \hat{x}_S^2 \hat{n}_S + 2 \hat{x}_S \hat{n}_S \hat{x}_S + \hat{n}_S \hat{x}_S^2 \rangle_{x_m} \right. \\ & \quad \left. + \delta x^2 \langle \hat{n}_S \rangle_{x_m} \right) P(x_m). \end{aligned} \quad (16)$$

The average expectation value of photon number after the measurement is given by

$$\langle \hat{n}_S \rangle_{\text{av}} = \int dx_m \langle \hat{n}_S \rangle_{x_m} P(x_m). \quad (17)$$

Using the index av to denote averages over expectation values after the measurement, the correlation $C(x_m^2; \langle \hat{n}_S \rangle_{x_m})$ may be expressed by the average final-state expectation values as

$$\begin{aligned} C(x_m^2; \langle \hat{n}_S \rangle_{x_m}) &= \left(\frac{1}{4} \langle \hat{x}_S^2 \hat{n}_S + 2 \hat{x}_S \hat{n}_S \hat{x}_S + \hat{n}_S \hat{x}_S^2 \rangle_{\text{av}} \right. \\ & \quad \left. - \langle n_S \rangle_{\text{av}} \langle x_S^2 \rangle_{\text{av}} \right). \end{aligned} \quad (18)$$

The correlation observed in the measurement is therefore given by a particular ordered product of operators. The most significant feature of this operator product is the $\hat{x}_S \hat{n}_S \hat{x}_S$ -term, in which the photon number operator \hat{n}_S is sandwiched between the field operators \hat{x}_S . The expectation value of $\hat{x}_S \hat{n}_S \hat{x}_S$ of an eigenstate of \hat{n}_S does not factorize into the eigenvalue of \hat{n}_S and the expectation value of \hat{x}_S^2 , because the field operators \hat{x}_S change the original state into a state with different photon number statistics. According to the commutation relations,

$$\hat{x}_S \hat{n}_S \hat{x}_S = \frac{1}{2} (\hat{x}_S^2 \hat{n}_S + \hat{n}_S \hat{x}_S^2) + \frac{1}{4}. \quad (19)$$

Therefore, the expectation value of $\hat{x}_S \hat{n}_S \hat{x}_S$ of a photon number state is exactly 1/4 higher than the product of the eigenvalue of \hat{n}_S and the expectation value of \hat{x}_S^2 . The correlation $C(x_m^2; \langle \hat{n}_S \rangle_{x_m})$ may then be expressed by the final-state expectation values as

$$C(x_m^2; \langle \hat{n}_S \rangle_{x_m}) = \left(\frac{1}{2} \langle \hat{x}_S^2 \hat{n}_S + \hat{n}_S \hat{x}_S^2 \rangle_{\text{av}} - \langle n_S \rangle_{\text{av}} \langle x_S^2 \rangle_{\text{av}} \right) + \frac{1}{8}. \quad (20)$$

Since the additional correlation of $\frac{1}{8}$ does not depend on the measurement resolution δx , it should not be interpreted as a result of the measurement dynamics. Instead, the derivation above reveals that it originates from the operator ordering in the quantum-mechanical expression for the correlation. Since it is the noncommutativity of operator variables that distinguishes quantum physics from classical physics, the contribution of $\frac{1}{8}$ is a nonclassical contribution to the correlation of photon number and fields. Specifically, it should be noted that the classical correlation of a well-defined variable with any other physical property is necessarily zero. Only the quantum-mechanical properties of noncommutative variables allow nonzero correlations of photon number and fields even if the field mode is in a photon number eigenstate. The operator transformation thus reveals that the correlation $C(x_m^2; \langle \hat{n}_S \rangle_{x_m})$ of $\frac{1}{8}$ found in measurements of the vacuum state is a directly observable consequence of the nonclassical operator order dependence of correlations between noncommuting variables.

V. EXPERIMENTAL REALIZATION: PHOTON-FIELD COINCIDENCE MEASUREMENTS

The experimental setup required to measure the correlation between a QND measurement of the quadrature component \hat{x}_S and the photon number after the measurement is shown in Fig. 1. It is essentially identical to the setups used in previous experiments [7,8]. However, instead of measuring the x quadrature in the output fields, it is necessary to perform a photon number measurement on the signal branch. The output of this measurement must then be correlated the output from the homodyne detection of the meter branch. The homodyne detection of the meter simply converts a high intensity light field into a current $I_M(t)$, while the signal readout produces discrete photon detection pulse. These pulses can also be described by a detection current $I_S(t)$, which should be related to the actual photon detection events by a response function $R_S(\tau)$, such that

$$I_S(t) = \sum_i R_S(t - t_i), \quad (21)$$

where t_i is the time of photon detection event i . According to the theoretical prediction discussed above, each photon number detection event should be accompanied by an increase of noise in the homodyne detection current of the meter. However, the temporal overlap of the signal current $I_S(t)$ and the increased noise in the meter current $I_M(t)$ is an important factor in the evaluation of the correlation. Due to the frequency filtering employed, the meter mode corresponding to a signal detection event is given by a filter function with a width approximately equal to the inverse frequency resolution of the filter. For a typical filter with a Lorentzian line-width of 2γ , the mode of interest would read

$$\hat{a}_i = \sqrt{\gamma} \int dt \exp(-\gamma|t-t_i|) \hat{a}(t). \quad (22)$$

The actual meter readout should therefore be obtained by integrating the current over a time of about $2/\gamma$. For practical reasons, it seems most realistic to use a direct convolution of the meter current I_M and the signal current I_S , adjusting the response function $R_S(\tau)$ to produce an electrical pulse of duration $2/\gamma$. A measure of the correlation $C(x_m^2; \langle \hat{n}_S \rangle_{x_m})$ can then be obtained from the current correlation

$$\xi C(x_m^2; \langle \hat{n}_S \rangle_{x_m}) = \overline{(I_S I_M)^2} - \overline{I_S^2} \overline{I_M^2}, \quad (23)$$

where the factor ξ denotes the efficiency of the measurement, as determined by the match between the response function $R_S(\tau)$ and the filter function given by Eq. (22). Moreover, the efficiency of the experimental setup may be reduced further by the limited quantum efficiency of the detector.

Fortunately, the requirement of efficiency for the experiment is not very restrictive, provided that the measurement resolution is so low that only few photons are created. In that case, the total noise average in the meter current I_M is roughly equal to the noise average in the absence of a photon detection event, which is very close to the shot noise limit of the homodyne detection. However, the fluctuations of the time averaged currents within a time interval of about $1/\gamma$ around a photon detection event in the signal branch correspond to the fluctuations of the measurement values x_m for a quantum jump event from zero photons to one photon. In particular, the measurement result $x_m(i)$ associated with a photon detection event at time t_i is approximately given by

$$x_m(i) \approx C \int dt R(t-t_i) I_M(t), \quad (24)$$

where C is a scaling constant which maps the current fluctuations of a vacuum input field onto an x_m variance of δx^2 . In the case of a photon detection event, however, the probability distribution over the measurement results $x_m(i)$ is given by the difference between the total probability distribution $P(x_m)$ and the part $P_0(x_m)$ of the probability distribution associated with no photons in the signal,

$$\begin{aligned} P_{QJ}(x_m) &= P(x_m) - P_0(x_m) \\ &= \langle \text{vac} | \hat{P}_{\delta x}^2 | \text{vac} \rangle - \langle \text{vac} | \hat{P}_{\delta x} | \text{vac} \rangle^2 \\ &= \frac{1}{\sqrt{2\pi(\delta x^2 + 1/4)}} \exp\left(-\frac{x_m^2}{2(\delta x^2 + 1/4)}\right) \\ &\quad - \sqrt{\frac{32\delta x^2}{\pi(1 + 8\delta x^2)^2}} \exp\left(-\frac{4}{1 + \delta x^2} x_m^2\right). \end{aligned} \quad (25)$$

Figure 3 shows the results for a measurement resolution of $\delta x = 1$, which is close to the experimentally realized resolution reported in [8]. There is only a slight difference in

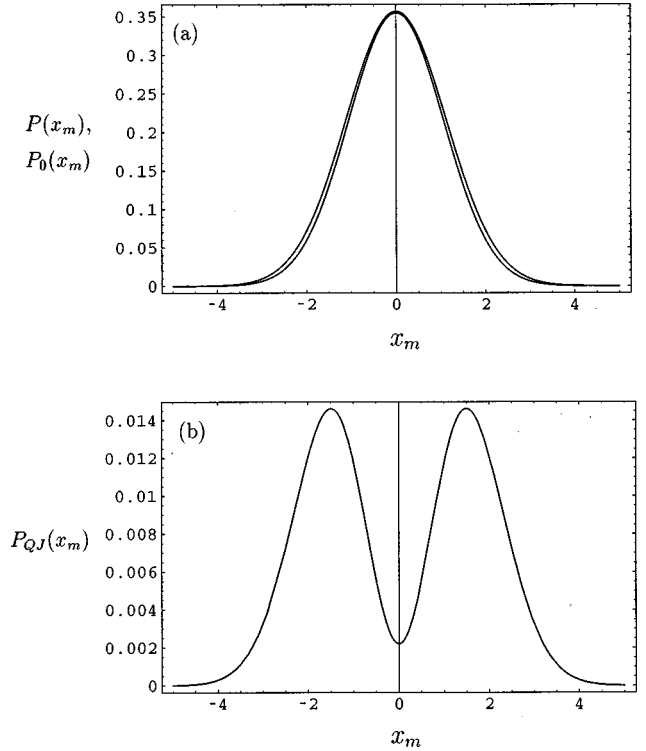


FIG. 3. Separation of the probability distribution $P(x_m)$ of the measurement result x_m into a component $P_0(x_m)$ associated with no quantum jump and a component $P_{QJ}(x_m)$ associated with a quantum jump to one or more photons at a measurement resolution of $\delta x = 1$. (a) shows both $P(x_m)$ and $P_0(x_m)$, which are only slightly different from each other. (b) shows the difference given by the quantum jump contribution $P_{QJ}(x_m)$. The total probability of a quantum jump at $\delta x = 1$ is 5.72%.

$P(x_m)$ and $P_0(x_m)$, even though the total probability of a quantum jump to one or more photons obtained by integrating $P_{QJ}(x_m)$ is about 5.72%. The peaks of the probability distribution are close to ± 2 , eight times higher than the fluctuation of \hat{x}_S in the vacuum. The measurement fluctuations corresponding to a photon detection event are given by

$$\frac{\int dx_m x_m^2 P_{QJ}(x_m)}{\int dx_m P_{QJ}(x_m)} = \frac{1}{4} + \delta x^2 \left(2 + \sqrt{1 + \frac{1}{8\delta x^2}} \right) \approx 3\delta x^2. \quad (26)$$

For $\delta x \gg 1$, this result is three times higher than the overall average. For $\delta x = 1$, the ratio between the fluctuation intensity of a detection event and the average fluctuation intensity of $1/4 + \delta x^2$ is still equal to 2.65. In other words, the fluctuations of the measurement result x_m nearly triple in the case of a quantum jump event. The corresponding increase in the fluctuations of the homodyne detection current I_M should be detectable even at low efficiencies ξ . Moreover, it does not matter how many photon events go undetected, since the ratio has been determined relative to the overall average of the meter fluctuations. It is thus possible to obtain experimental evidence of the fundamental correlation of field com-

ponent and photon number even with a rather low overall efficiency of the detector setup.

VI. INTERPRETATION OF THE QUANTUM JUMP STATISTICS

What physical mechanism causes the quantum jump from the zero photon vacuum to one or more photons? The relationship between the photon number operator and the quadrature components of the field is given by

$$\hat{n}_S + \frac{1}{2} = \hat{x}_S^2 + \hat{y}_S^2. \quad (27)$$

According to Eq. (2) describing the measurement interaction, the change in photon number \hat{n}_S should therefore be caused by the change in \hat{y}_S caused by \hat{y}_M ,

$$\hat{U}_{SM}^{-1} \hat{n}_S \hat{U}_{SM} = \hat{n}_S - 2f \hat{y}_S \hat{y}_M + f^2 \hat{y}_M^2. \quad (28)$$

Thus the change in the photon number does not depend explicitly on either the measured quadrature \hat{x}_S or the meter variable \hat{x}_M . Nevertheless, the meter readout shows a strong correlation with the quantum jump events. In particular, the probability distribution of meter readout results x_m for a quantum jump to one or more photons shown in Fig. 3 has peaks at values far outside the range given by the variance of the vacuum fluctuations of \hat{x}_S .

Moreover, the correlation between the readout and photon number after the measurement does not disappear in the limit of low resolution ($\delta x \rightarrow \infty$). Rather, it appears to be a fundamental property of the vacuum state even before the measurement. This is confirmed by the operator formalism, which identifies the source of the correlation as the expectation value $\langle \hat{x}_S \hat{n}_S \hat{x}_S \rangle$. This expectation value is equal to $\frac{1}{4}$ in the vacuum, even though the photon number is zero. Since the operator formalism does not allow an identification of the operator with the eigenvalue unless it acts directly on the eigenstate, it is possible to find nonzero correlations even if the system is in an eigenstate of one of the correlated variables. In particular, the action of the operator \hat{x}_S on the vacuum state is given by

$$\hat{x}_S |\text{vac}\rangle = \frac{1}{2} |n_S = 1\rangle, \quad (29)$$

so the operator \hat{x}_S which should only determine the statistical properties of the state with regard to the quadrature component x_S changes the vacuum state into the one-photon state. The application of operators thus causes fluctuations in a variable even when the eigenvalue of that variable is well defined.

The nature of this fluctuation might be clarified by a comparison of the nonclassical correlation obtained for fields and photon number in the vacuum with the results of quantum tomography by homodyne detection [12,13]. In such measurements, the photon number is never obtained. Rather, the complete Wigner distribution $W(x_S, y_S)$ can be reconstructed

from the results. It is therefore possible to deduce correlations between the field components and the field intensity defined by $I = x_S^2 + y_S^2$, which is the classical equivalent of Eq. (27). For the vacuum, the Wigner function reads

$$\int dx_S dy_S x_S^4 W_0(x_S, y_S) - \left(\int dx_S dy_S x_S^2 W_0(x_S, y_S) \right)^2 = \frac{1}{8}. \quad (30)$$

The correlation of I and x_S^2 is given by

$$\begin{aligned} C(x_S^2; I) &= \int \left(\int dx_S dy_S x_S^2 I W_0(x_S, y_S) \right) \\ &\quad - \left(\int dx_S dy_S x_S^2 W_0(x_S, y_S) \right) \\ &\quad \times \left(\int dx_S dy_S I W_0(x_S, y_S) \right) \\ &= C(x_m^2; \langle n_S \rangle_{x_m}) = \frac{1}{8}. \end{aligned} \quad (31)$$

Thus, the correlation between $I = x_S^2 + y_S^2$ and x_S^2 described by the Wigner distribution is also equal to $\frac{1}{8}$. In fact, the ‘‘intensity fluctuations’’ of the Wigner function can be traced to the same operator properties that give rise to the correlations between the field measurement result and the induced photon number. For arbitrary signal fields, the correlation between the squared measurement result and the photon number after the measurement can therefore be derived by integrating over the Wigner function of the signal field after the measurement interaction according to Eq. (31).

Of course the ‘‘intensity fluctuations’’ of the Wigner function cannot be observed directly, since any phase-insensitive determination of the photon number will reveal the well-defined result of zero photons in the vacuum. Nevertheless even a low resolution measurement of the quadrature component \hat{x}_S which leaves the vacuum state nearly unchanged reveals a correlation of \hat{x}_S^2 and n_S which corresponds to the assumption that the measured quadrature \hat{x}_S contributes to a fluctuating vacuum energy. The quantum jump itself appears to draw its energy not from the external influence of the measurement interaction, but from the fluctuating energy contribution \hat{x}_S^2 . These energy fluctuations could be interpreted as virtual or hidden fluctuations existing only potentially until the energy uncertainty of the measurement interaction removes the constraints imposed by quantization and energy conservation. In particular, energy conservation does require that the energy for the quantum jump is provided by the optical parametric amplification process. Certainly the *average* energy is supplied by the pump beam. However, the energy content of the pump beam and the meter beam cannot be defined due to the uncertainty principle. The pump must be coherent and the measurement of the meter field component \hat{x}_M prevents all energy measurements in that field. If it is accepted that quantum-mechanical reality is somehow conditioned by the circumstances of the measurement, it can be argued that the reality of the quan-

tized photon number only exists if the energy exchange of the system with the environment is controlled on the level of single quanta. Otherwise, it is entirely possible that the vacuum energy might not be zero as suggested by the photon number eigenvalue, but might fluctuate according to the statistics suggested by the Wigner function.

Even though it may appear to be highly unorthodox at first, this “relaxation” of quantization rules actually corresponds to the noncommutativity of the operators, and may help explain the seemingly nonlocal properties of entanglement associated with the famous EPR paradox [14]. The definition of elements of reality given by EPR reads “*If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*” This definition of elements of reality assumes that the eigenvalues of quantum states are real even if they are not confirmed in future measurements. In particular, the photon number of the vacuum would be considered as a real number, not an operator, so the operator correlation $\langle \hat{x}_S \hat{n}_S \hat{x}_S \rangle$ should not have any physical meaning. However, the nonzero correlation of fields and photon number in the vacuum observed in the QND measurement discussed above suggests that *even the possibility of predicting the value of a physical quantity with certainty only defines an element of reality if this value is directly observed in a measurement.* Based on this conclusion, there is no need to assume any “spooky action at a distance,” or physical nonlocality, in order to explain Bell’s inequalities [15]. Instead, it is sufficient to point out that knowledge of the wave function does not provide knowledge of the type of measurement that will be performed. In the case of spin- $\frac{1}{2}$ systems, the quantized values of spin components are not a property inherent in the spin system, but a property of the measurement actually performed. To assume that spins are quantized even without a measurement does not correspond to the implications of the operator formalism, since it is not correct to replace operators with their eigenvalues.

In the same manner, the correlation discussed in this paper would be paradoxical if one regarded the photon number

eigenvalue of zero in the vacuum state as an element of reality independent of the measurement actually performed. One would then be forced to construct mysterious forces changing the photon number in response to the measurement result. However, the operator formalism suggests no such hidden forces. Instead, the reality of photon number quantization depends on the operator ordering and thus proofs to be rather fragile.

VII. SUMMARY AND CONCLUSIONS

The change in photon number induced by a quantum non-demolition measurement of a quadrature component of the vacuum is strongly correlated with the measurement result. An experimental determination of this correlation is possible using optical parametric amplification in a setup similar to previously realized QND measurements of quadrature components [7,8]. The observed correlation corresponds to a fundamental property of the operator formalism which allows nonvanishing correlations between noncommuting variables even if the system is in an eigenstate of one of the variables.

The quantum jump probability reflects the properties of intensity fluctuations corresponding to the vacuum fluctuations of the field components. The total correlation of fields and photon number therefore reproduces the result that would be expected if there was no quantization. It seems that quantum jumps are a mechanism by which the correspondence between quantum mechanics and classical physics is ensured. The quantum jump correlation observable in the experimental situation discussed above thus provides a link between the discrete nature of quantized information and the continuous nature of classical signals. Finite resolution QND measurements could therefore provide a more detailed understanding of the nonclassical properties of quantum information in the light field.

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